Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

TRANSVERSE FLOW IN A RIVER DUE TO EARTH'S ROTATION

By Hans A. Neu[1]

INTRODUCTION

Any object moving over the surface of the earth experiences a transverse force normal to its appointed path—to the right in the northern hemisphere, to the left in the southern hemisphere. The magnitude of this force is directly proportional to the velocity of the moving object; it is greatest near the Poles, and decreases to zero at the Equator.

The phenomenon was first described in 1835 by G. G. Coriolis, after whom it is named. It is described herein only briefly and relevant to the purpose of the investigation. For a more detailed and exact description, reference is made to textbooks of physics, oceanography,[2] and meteorology.[3]

CORIOLIS FORCE

The Coriolis force is an inertial force, representing the inertial reluctance of the moving body to participate in the rotational motion of the planet.

Qualitatively, Faller and von Arx[4] describe it with regard to north-south and east-west planes on the northern hemisphere as follows:

> "For a particle moving in the north-south direction such motion towards the right is a manifestation of the conservation of absolute angular momentum $\Omega^2 R$ = constant (notations from paper), in which Ω is the particle's absolute angular velocity and R is its radius from the axis of

Note.—Discussion open until February 1, 1968. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 93, No. HY5, September, 1967. Manuscript was submitted for review for possible publication on November 30, 1966.

[1] Senior Research Officer, National Research Council of Canada, Ottawa, Ont.

[2] Krümmel, O., and Ludin, A., Handbuch der Ozeanographie, Bd. II, J. Engelhorn, Stuttgart, Germany, 1911.

[3] Mügge, R., "Wetterkunde und Wettervorhersage," Die Welt im Fortshritt, Reihe 1, Bd. 6, Berlin, Germany, 1936.

[4] Faller, A. J., and von Arx, W. S., "The Modelling of Fluid Flow on a Planetary Scale," Contribution No. 987, Woods Hole Oceanographic Institution, 1959.

rotation. Thus if R decreases, as in the case for northward motion on the earth, Ω must increase and a component of motion towards the east is apparent. Relative motion towards the west occurs where the opposite is true, or for southward motion, where R increases and Ω decreases. Curvature to the right for a particle moving in the east-west plane is not ordinarily explained but the apparent force to the right is to be due to an excess or deficiency of centrifugal forces due to the rotation about the earth's axis."

Analytically, the ordinary Newtonian laws of motion derived for a rectangular coordinate system at rest can be applied provided the Coriolis accelerations in the directions of the coordinate axes are added. These are

$$a_x = -2\omega \sin\phi \, v_y \quad \text{(1a)}$$

$$a_y = 2\omega \sin\phi \, v_x \quad \text{(1b)}$$

and

$$a_z = 2\omega \cos\phi \, v_z \quad \text{(1c)}$$

in which a_x, a_y, a_z are the accelerations, and v_x and v_y the velocity components in the directions of the axes X, Y, and Z which are alined in the directions of the mean flow, transversely and vertically, respectively. The angular velocity of the earth $\omega = 2\pi/86{,}164$ sec $= 7.29 \times 10^{-5}$ sec^{-1}, and ϕ is the geographical latitude.

The vertical component a_z, which acts in the same direction as the gravitational acceleration g is, for water, too small to be considered; consequently, only the horizontal components of the Coriolis force are taken into account.

In meteorology and in oceanography, this Coriolis force is considered one of the main forces influencing large-scale atmospheric and oceanic motions. It has been argued, however, that in the confined waters of channels, rivers and narrow lakes it would have no influence on the flow paths, the presence of banks preventing lateral deviation.

In principle, this is true. However, in 1860 von Baer[5] observed that depths on the right side (viewing downstream) of large North Siberian rivers were greater and the banks steeper than those on the left side. He concluded that this deformation was caused by pressure from the Coriolis force exerted against the right hand banks of the river channels. At the turn of the century, this phenomenon was also observed on the Yukon River by Eakin[6] although he concluded that a transverse circulation was probably the cause. In 1938, Wittman and Böss[7] while studying the dynamics of flow in bends, demonstrated that in bends of alluvial channels a transverse circulation, initiated by the difference between centrifugal forces in the upper and lower layers, causes the deformation of the bed and banks. This transverse circulation has been studied extensively in the meantime and theories developed which, though approximate, were verified by field and model investigations.

Transverse flow in straight channels, due to the effect of the earth's rotation, is similar in every respect to that in bends. The basic assumptions

[5] von Baer, K. E., "Uber ein Allgemeines Gesetz in der Gestaltung der Flussbetten," Bulletin, Academie des Sciences de Petersburg, 1860.

[6] Eakin, H. M., "The Influence of the Earth's Rotation Upon the Lateral Erosion of Streams," Journal of Geology, Vol. 18, No. 5, Aug., 1910, pp. 435-447.

[7] Wittman, H., and Böss, P., Wasser- und Geschiebebewegung in gekrümmten Flusstrecken, Julius Springer, Berlin, Germany, 1938.

made to define and develop the laws for explaining the flow in bends are valid also for explaining the flow in straight channels under the influence of the earth's rotation. Van Bendegom[8] was the first to apply this principle. The theory is as rational as that developed for flow in bends, but, while the circulation of the flow in a bend could be observed and measured, the existence of a similar circulation due to the earth's rotation could not be proved or demonstrated. This lack of evidence was the main reason for the uncertainty which prevailed regarding its existence or significance. In confined waters, it was generally assumed to be negligible.

Transverse motion in a straight section of the St. Lawrence River, thought to have been initiated at least in part by the rotation of the earth, was first observed accidentally during a survey between Montreal and Sorel in 1959. A survey in 1962 was planned to measure it in more detail. Both surveys were conducted exclusively in long, wide, straight, uniform river channels where steady flow conditions prevailed.

No measurements were made of the circulation in the estuary, where the longitudinal slope of the water surface gradually decreases and tidal and density currents govern the flow regime. Under these circumstances, the earth's rotation assumes a still more important role, frequently forming ebb and flood channels in the alluvial bed, as observed by Hensen[9] in 1939.

Since the existence of such a transverse circulation appears to have been demonstrated by the surveys, a more rational explanation is possible of phenomena unique to large rivers, estuaries, and open waters such as landlocked seas and large lakes which are part of river systems such as the St. Lawrence.

THEORY

The acceleration due to the Coriolis effect which acts at right angles to the motion of an object or particle of matter moving over the earth is

$$a = 2\omega \sin\phi\,(v) \qquad (2)$$

Under the influence of this acceleration, a water particle moving with a constant horizontal velocity v, subject to no other forces in the horizontal plane, follows a circular path known as the "inertia circle," having the radius

$$r = \frac{v}{2\omega \sin\phi} \qquad (3)$$

Where the particle is confined in a straight channel, it cannot follow the path defined by Eq. 3 and a force must arise from the opposing bank of the same magnitude as the Coriolis force but acting in the opposite direction. This opposing force is provided by an increased water level along the right bank (viewed downstream) of channels in the northern hemisphere. The force per unit mass is

$$p = 2\omega \sin\phi\, v_o \qquad (4)$$

in which v_o = the mean axial velocity over an axial vertical plane of the moving

[8] Van Bendegom, L., "Einige beschouwingen over riviermorphologie en rivierverbetering," De Ingenieur, 59(4), B.1-11, 1947.

[9] Hensen, W., "Der Einfluss der Erdumdrehung auf Tideflüsse in der Natur und im Model," Die Bautechnik, Heft 21, Berlin, Germany, 1939.

body of water. The transverse slope of the water surface to maintain the flow along a straight line is

$$\tan \delta = \frac{2\omega \sin \phi \, v_o}{g} \quad (5)$$

in which δ = the angle of inclination of the water surface with the horizontal plane.

Accordingly, the Coriolis acceleration in an idealized straight uniform frictionless channel, having uniform velocity from the surface to the bed, would be balanced at all levels by the forces of the pressure gradient of the transverse slope. Under such circumstances, no transverse motion could develop.

In natural channels, however, the axial velocities over an axial vertical plane are not uniform, being greater than the average near the surface and smaller near the bed due to the presence of boundary friction. If v is the velocity in a particular layer of the flow and v_o the mean velocity, then

$$v = v_o e \quad (6)$$

in which e represents the ratio between the two velocities.

Introducing Eq. 6 into Eq. 2, the equation for the Coriolis acceleration for a particular layer or depth becomes

$$a = 2\omega \sin \phi \, (e v_o) \quad (7)$$

Since e varies from a value less than 1 near the bed to a value greater than 1 near the surface, the Coriolis accelerations, according to Eq. 7, must vary likewise. As shown in Eq. 4, the opposing transverse pressure due to the transverse slope in the water surface is only related to the mean velocity v_o. The Coriolis force is, therefore, greater than the opposing pressure force near the surface and smaller near the bed. The differential force is in the direction of the Coriolis force near the surface and against it near the bed, thus creating a state of unbalance, and in turn a transverse current.

This transverse current in turn develops a friction slope which is added to the transverse slope of the free water surface. The combined transverse water slope S_y attained under a state of equilibrium may be written

$$S_y = \beta \, \frac{2\omega \sin \phi \, v_o}{g} \quad (8)$$

in which $\beta > 1$ related to the boundary friction.

When analyzing the motions occurring in a straight and relatively wide channel with a free surface and uniform flow, three forces must be considered: gravity, Coriolis, and friction. For dynamic equilibrium, the following equations in Cartesian coordinates must be valid, i.e.,

$$g S_x - 2\omega \sin \phi \, v_y - F_x = 0 \quad (9a)$$

and

$$-g S_y + 2\omega \sin \phi \, v_x - F_y = 0 \quad (9b)$$

in which S = the slope of the water surface, and F = the frictional force per unit mass for turbulent motion. As already mentioned, the third term of the dynamic equation and the forces deriving from the vertical velocity component are neglected. It can be assumed, as will be shown later, that in a wide channel away from the walls or banks the vertical velocity component v_z is small compared with the horizontal components v_x and v_y. Furthermore, the axial component of the Coriolis force due to transverse flow has an insignificant influence on the flow in the X-direction and can also be omitted. The force diagram is shown on Fig. 1.

Eq. 9(a) then becomes

$$gS_x - F_x = 0 \qquad (10)$$

which is the equation for uniform flow in open channels.

Each of the forces in the equations varies with distance above the bed. In a channel of depth d, the axial and transverse components of the gravitational

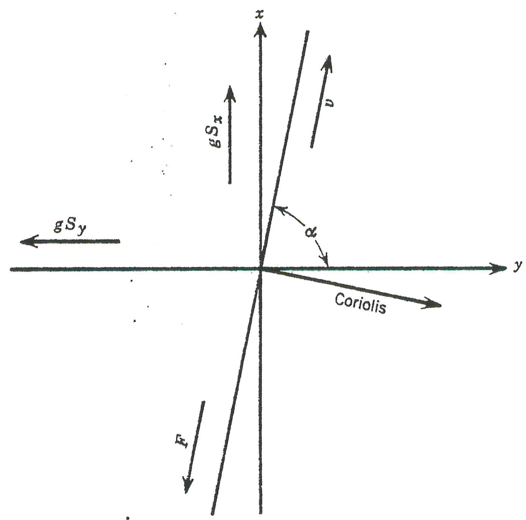

FIG. 1.—DIAGRAM OF MAIN FORCES

force per unit mass, $G(z)$, at the height z above the bed of the channel can be expressed as

$$\int_z^d G_x\, dz = \rho g d S_x \left(1 - \frac{z}{d}\right) \qquad (11a)$$

and

$$\int_z^d G_y\, dz = -\rho g d S_y \left(1 - \frac{z}{d}\right) \qquad (11b)$$

and the transverse component $C_y'(z)$ of the Coriolis force per unit mass at the same elevation is

$$\int_z^d C_y'\, dz = \rho\, 2\, \omega \sin \phi \int_z^d v_x\, dz \qquad (12)$$

The frictional stresses due to turbulent motion may be estimated by applying Prandtl's "mixing length" theory. The eddy viscosity

$$\epsilon = \rho\, \ell^2 \frac{\partial v}{\partial z} \qquad (13)$$

is assumed to be independent of direction in the x-y plane; ℓ is Prandtl "mixing length." The effective stress components per unit mass then are

$$F_x = -\rho\,\ell^2 \left(\frac{\partial v_x}{\partial z}\right)^2 \quad \text{(14}$$

and $$F_y = -\rho\,\ell^2 \frac{\partial v_x}{\partial z}\,\frac{\partial v_y}{\partial z} \quad \text{(14}$$

Substituting for the terms of Eqs. 9(b) and 10 those derived in Eqs. 11, and 14, the dynamic equation now reads

$$\rho\,g\,d\,S_x\left(1 - \frac{z}{d}\right) - \rho\,\ell^2\left(\frac{\partial v_x}{\partial z}\right)^2 = 0 \quad \text{(15}$$

$$-\rho\,g\,d\,S_y\left(1 - \frac{z}{d}\right) + \rho\,2\omega\sin\phi\int_z^d v_x\,dz - \rho\,\ell^2\frac{\partial v_x}{\partial z}\,\frac{\partial v_y}{\partial z} = 0 \quad \text{(15}$$

To obtain expressions describing the axial and transverse flow and transverse surface slope, the equations require a number of analytical tran formations and integrations. Rozovskii,[10] whose study of the transverse fl in bends is based on an inertia effect through curvature similar to that i straight channel due to the earth's rotation, made the necessary mathemati transformations and arrangements. For the complete development of equations reference is made to his work. For this investigation, the equati derived for transverse flow in bends are applicable also to transverse flow straight channels due to the earth's rotation.

Rozovskii used the logarithmic form to represent the vertical velocity d tribution of the axial flow, and the Chezy formula to express the roughness the channel and the slope of the water surface.

His results are as follows:

The axial velocity component is

$$v_x(\eta) = v_o\left[1 + \frac{\sqrt{g}}{k_o\,C}(1 + \ln\,\eta)\right] \quad \text{(}$$

The coefficient β of the transverse surface slope is

$$\beta = 1 + \frac{g}{k_o^2\,C^2} \quad \text{(}$$

The transverse velocity component is

$$v_y(\eta) = v_o\,\frac{d}{r_c}\,\frac{1}{k_o^2}\left[E_1(\eta) - \frac{\sqrt{g}}{k_o\,C}\,E_2(\eta)\right]$$

The deviation at any point along the vertical is

[10] Rozovskii, I. L., "Flow of Water in Bends of Open Channels," Academy of Scie of the Ukrainian SSR, Kiev, USSR, 1957; published for the National Science Founda and the U. S. Dept. of the Interior, Washington, D. C., by the Israel Program for Sc tific Translations, 1961.

$$\tan \alpha = \frac{d}{r_c} \frac{1}{k_o^2} \left[\frac{E_1(\eta) - \dfrac{\sqrt{g}}{k_o C} E_2(\eta)}{1 + \dfrac{\sqrt{g}}{k_o C} (1 + \ln \eta)} \right] \quad \text{. . . . . (19)}$$

In these equations, $v_x(\eta)$ and $v_y(\eta)$ = the axial and transverse velocity components at the relative or dimensionless height $\eta = z/d$, C = the Chezy coefficient, k_o = the von Karman constant, r_c = the radius of curvature of the bend, α = the angle of deviation from the axial direction of flow, and

$$E_1(\eta) = \int \frac{2 \ln \eta}{\eta - 1} d\eta \quad \text{. . . . . (20a)}$$

$$E_2(\eta) = \int \frac{\ln^2 \eta \, d\eta}{\eta - 1} \quad \text{. . . . . (20b)}$$

are functions, the values of which can be obtained from graphs on Fig. 2 (from Rozovskii[10]).

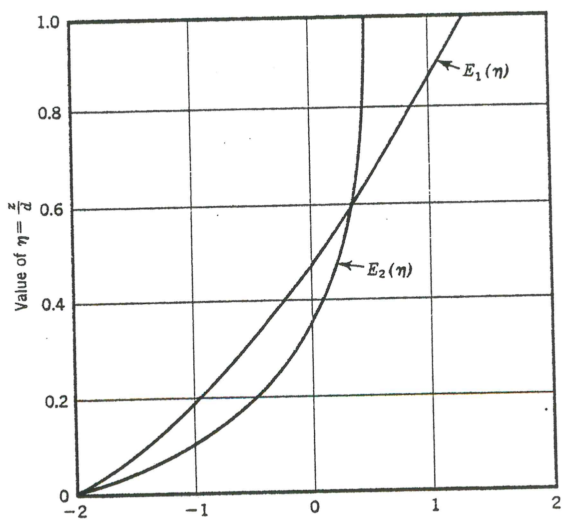

FIG. 2.—FUNCTIONS $E_1(\eta)$ AND $E_2(\eta)$

As indicated by the above reasoning, a stream flowing in a straight section of channel under the influence of the earth's rotation behaves as if it were flowing around a curve on a nonrotating planet, having the radius of the inertia circle

$$r = \frac{v_o}{2 \omega \sin \phi} \quad \text{. . . . . (21)}$$

Substituting the radius of the inertia circle for r_c of the curvature in Eq. 19 gives

$$\tan \alpha = \frac{d\ 2\omega \sin \phi}{v_o} \frac{1}{k_o^2} \left[\frac{E_1(\eta) - \frac{\sqrt{g}}{k_o C} E_2(\eta)}{1 + \frac{\sqrt{g}}{k_o C} (1 + \ln \eta)} \right] \quad \text{. . . . . (22)}$$

APPLICATION OF THE THEORY

A major restriction to the use of the theory, which is as important to flow in straight channels under the influence of the earth's rotation as to flow around bends, is that it applies only to flow in wide channels having certain characteristics. For flow around bends, Rozovskii[10] classified these channels by two

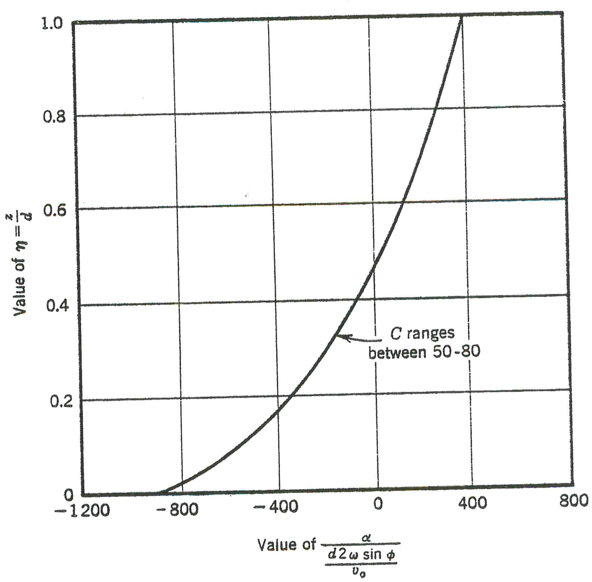

FIG. 3.—DISTRIBUTION OF FLOW DIRECTION ACCORDING TO EQ. 22 (NOTE: α IN DEGREES)

ratios, (1) depth over width, d/b, and (2) depth over radius of curvature, d/r_c, both of which should be less than 0.1 for the theory to apply. The flow in channels having larger ratios is still more complex, due to the effect of banks or walls and the inertial forces of the transverse flow. These latter forces can be neglected in relatively wide, shallow channels but are important in deep channels.

The ratio $d/b < 0.1$ applies to straight channels as it does to curved, whereas the ratio d/r_c which expresses the curvature of the bend, for cross current to develop fully, may be replaced by the ratio of depth to radius of the inertia circle $(d\ 2\,\omega \sin \phi)/v_o$. There is, however, no way to derive the significant emmagnitude of the second ratio except from model studies. Until such an empirical value has been obtained, it appears advisable to use values of this ratio not larger than 0.02 to 0.03, to be certain of preventing its application to channels which are too deep.

The ratios of the angles between the directions of flow and the channel axis to the inertia circle are plotted on Fig. 3. The values are derived from Eq. 22 using Chezy coefficients ranging from 50 to 80. The results for different values of the Chezy coefficients differed so little that it was considered sufficient to represent the entire range by a single curve. This indicates that the effect of roughness of the bed is small.

As expected and demonstrated with this flow direction distribution curve, the layers most affected by the transverse circulation are those at the surface and near the bed; the deviation of the surface layer being approximately half as much as that of the layer near the bed. The magnitude of the deviations of these two layers is generally sufficient to evaluate the significance of the transverse flow with regard to the main flow.

For the surface layer and for the layer near the bed, the term

$$\frac{1}{k_o^2}\left[\frac{E_1(\eta) - \dfrac{\sqrt{g}}{k_o C} E_2(\eta)}{1 + \dfrac{\sqrt{g}}{k_o C}(1 + \ln \eta)}\right]$$

on the right side of Eq. 22 is, for practical purposes, nearly a constant value. The deviations of these layers can, therefore, be expressed by

$$\tan \alpha_S = D_S \frac{d\ 2\omega \sin \phi}{v_o} \qquad (23a)$$

and

$$\tan \alpha_B = D_B \frac{d\ 2\omega \sin \phi}{v_o} \qquad (23b)$$

in which D is a constant, and subscripts S and B denote surface and bed layer, respectively.

For the range of angles over which these approximations apply, and for latitudes having Coriolis parameters of approximately 1×10^{-4} (the parameter varies from 0.73×10^{-4} to 1.37×10^{-4} between the 30° and 70° North and South Latitude, the area of the globe where most of the continents with rivers, except Africa and part of South America, are located), Eqs. 23 can be still further simplified and the deviation angles directly expressed in degrees as

$$\alpha_S = 0.036 \frac{d}{v_o} \qquad (24a)$$

and

$$\alpha_B = 0.060 \frac{d}{v_o} \qquad (24b)$$

These simplified approximations show that the deviation angles depend primarily on depth and velocity. They also show under which circumstances the deviations are measurable. If 1° to 2° deviation is the order of magnitude which can be measured by instruments in the field—a total difference between surface and bed, for example, of 3° to 4°—the velocities at which these angles would occur in the channels 50 m (150 ft), 20 m (60 ft), and 5 m (15 ft) deep would be 143 cm per sec (4.7 fps), 57 cm per sec (1.87 fps), and 14 cm per sec (0.47 fps), respectively.

These examples indicate that velocities must be relatively small compared with the depth of the channels for deviations of significant size to occur. This requirement divides natural channels into two groups; one in which the deviation cannot be measured because the angles are too small, and the other in which deviations are significant. The first group is represented by sediment-bearing rivers which require high current velocities to transport material. Because the numerical value of the ratio d/v_0 of this type of river is nearly always less than 20, the deviations can hardly be greater than 1° to 2° except at their mouths and estuaries. Nevertheless, this does not mean that the transverse motion resulting from these small deviations has no morphological significance, for it is well established that, in geological terms, these channels migrate transversely.

The second group includes deep, wide rivers and river pool sections such as storage and power lakes where $d/v_0 > 20$ and appreciable deviations can occur. For example, at the lower end of Lake Ontario, where the flow is compressed into channels having depths greater than 30 m (100 ft) and average velocities less than 10 cm per sec (0.3 fps), the sum of the surface and bed layer deviations should be greater than 25°, according to Eqs. 24. Farther downstream, at the site of the survey, the sum should be between 7° and 8°, while in Lake St. Lawrence, a river pool formed in 1958 as part of the St. Lawrence power and navigation scheme, the sum should be more than 12°.

SURVEY

The section chosen for the survey was located on the St. Lawrence River between Brockville and Prescott, about 80 km (50 miles) from the outlet of Lake Ontario, where the river is uniform and relatively straight for more than 24 km (15 miles). The channel is 1,500 m (5,000 ft) wide and 25.3 m (83 ft) deep with an axial surface slope during the survey of less than 1×10^{-6}. The Coriolis parameter, $2\omega \sin \phi$, at the latitude of the survey, 44° 40' N, is 1.025×10^{-4}. The velocity and direction of the current were measured with Ott directional current meters at 1.5 m (5 ft) depth intervals, the arithmetical mean of five measurements being taken.

The location of the survey positions and the properties measured are shown on Fig. 4. Positions 1 and 6 were affected, obviously, by large crosswise gradients in the river bed and by the shore, but the velocities in the main portion of the river, except those of position 5, were quite regular. The surface velocities of positions 2, 3, 4 and 5 ranged from 30.5 cm per sec to 32 cm per sec (1.0 fps to 1.1 fps); the ratios between the axial surface velocity and the mean velocity varied only from 1.20 to 1.27, although position 5 differed with a ratio of 1.07. The deviation of the current from the axis of the channel was to the right on the surface and to the left on the bed, facing downstream. The greatest deviation was found to occur at position 2, where the surface water digressed 15° to the right and the bed layer the same amount to the left. In the center of the river, at positions 3 and 4, the deviations were smaller, being on the order of 8° on the surface and 12° to 15° near the bed. For position 5, the results at first appeared random; however, plotting revealed that with the exception of local disturbances there was no general deviation of the flow at this section.

The transverse velocity components computed from the deviations of the

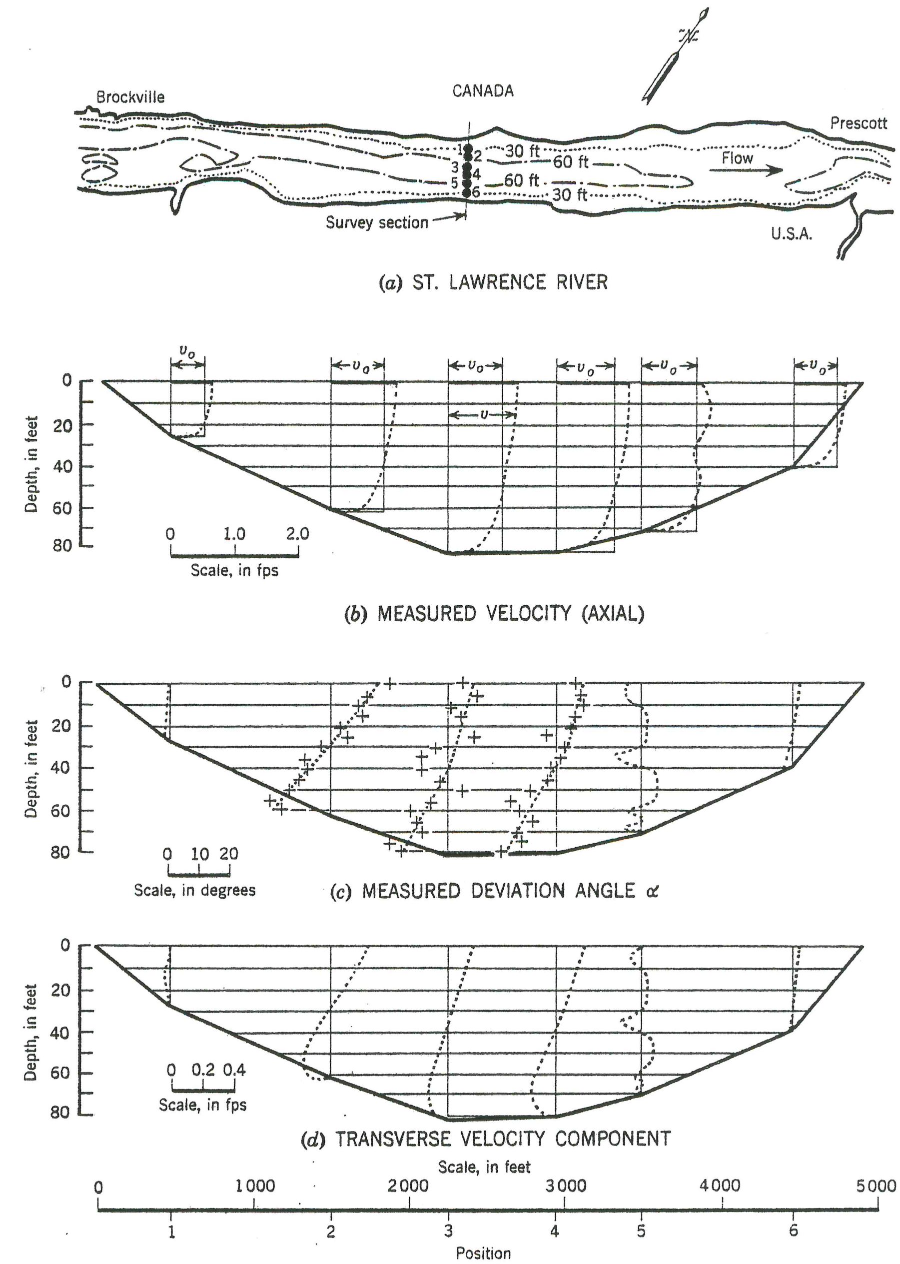

FIG. 4.—MAP OF THE ST. LAWRENCE RIVER AND SURVEY DATA

recorded velocities are shown also on Fig. 4. The strength of the cross current was found to be between 4.9 cm per sec (0.16 fps) and 7.3 cm per sec (0.24 fps) to the right at the surface, and between 2.7 cm per sec (0.09 fps) and 5.2 cm per sec (0.17 fps) to the left at the bed. The cross current also was greatest at position 2, while it was practically nonexistent at position 5.

Generally, it can be stated that there is a transverse circulation in most of the river section which overturns the water masses forming a right-hand or clockwise spiral flow. The circulation obviously was shifted from the center of the channel toward the left.

COMPARISON OF THEORY AND OBSERVATION

First, the roughness characteristic of the channel as indicated by the profiles of the measured velocities will be evaluated briefly and the shape of these curves compared with the logarithmic form, used by Rozovskii[10] for the theory.

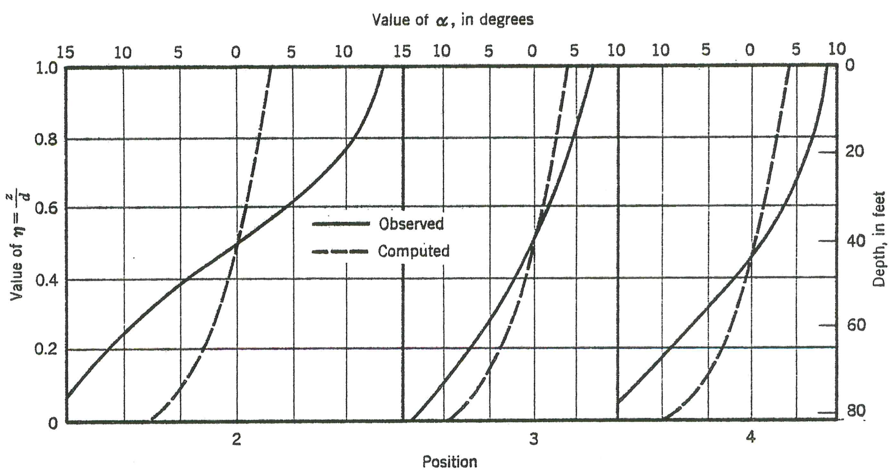

FIG. 5.—OBSERVED AND COMPUTED DEVIATION ANGLES IN THE ST. LAWRENCE RIVER

The roughness length k, which is related to the height of the irregularities at the bed, or, more conveniently, its relative value d/k, is a measure of the roughness in the channel. Its value can be determined from the velocity gradient of the measured profiles by applying the logarithmic law, describing the character of the turbulence in the flow. At positions 2, 3, and 4 the values of d/k were between 5 and 10, indicating that the bed of the channel is rather rough. At positions 1, 5, and 6 the velocity profiles (Fig. 4) did not comply with the logarithmic distribution, having a more rectangular shape with a narrow zone of velocity decay near the bottom. There are several reasons for this, one of which is the proximity of the shore, particularly at positions 1 and 6, and the effect of the transverse gradient of the bed on the flow. Another is that there is a vertical velocity component in these zones due to the transverse circulation, which gains strength toward the sides and modifies the axial velocity profiles to those observed. There were no noticeable cross currents at positions 1, 5 and 6. The deviation angles computed for positions 2, 3 and 4 are plotted on Fig. 5 together with those observed in the river. The Chezy coefficient C required for Eq. 22 is derived from the Chezy formula

$$v = C \sqrt{d\ S_x} \qquad (25)$$

its numerical value in this part of the St. Lawrence River being approximately 70. It is evident that the plots of observed and computed angles are similar in shape and in their points of zero deviation; however, the computed angles are one-half of those measured at positions 3 and 4, and less than one-third those at position 2. Therefore, the results agree qualitatively, but differ quantitatively by a factor of 2 to 3.

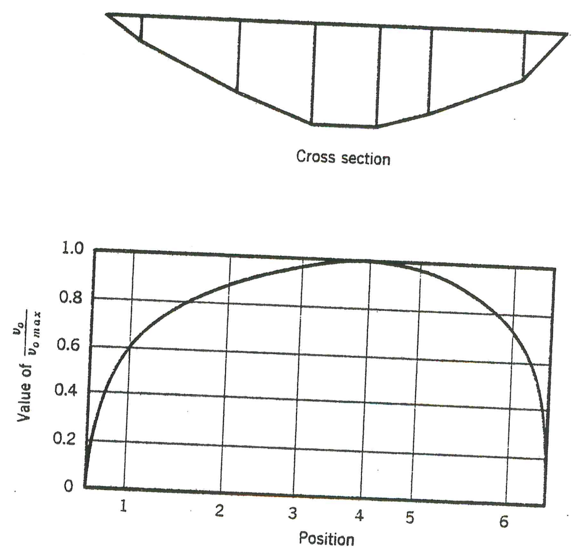

FIG. 6.—DISTRIBUTION OF AXIAL MEAN VELOCITY ACROSS CHANNEL

Although this difference may appear excessive, there are several reasons to account for it. The first is that the theoretical results are quantitatively too small. Rozovskii[10] also came to this conclusion in his investigation for flow in bends. He states that the results from theoretical formulas are not exaggerated but underestimated. His statement is based on a comparison of theoretical results with data from a number of model and field observations. Before a similar statement can be made for flow deviation in straight channels due to the earth's rotation, observations are required in addition to those submitted herein.

The second reason for the difference is that the river channel, particularly in its deep section, is not perfectly straight but has a slight bend to the left which imposes a clockwise circulation on the flow, like that due to the earth's rotation. According to the theory for flow around bends, however, the magnitude of this deviation is small, the value of α amounting only to approximately 1°.

The third reason, which some may consider speculative, has in recent years increasingly occupied the attention of researchers in the field of secondary motions, channel formation, and sediment transport. In natural channels the flow is confined by the shore and the bed, which usually slopes toward the center of the channel. The cross section of the St. Lawrence River where the survey was conducted is a polygon or a wide parabola. Owing to the boundary-shear turbulence initiated by the friction of the bed and sides of the channel, the water velocities are different at various altitudes above the bed. These variations exist not only vertically but also transversely. As shown on Fig. 6, which is a plot of the ratios between the average velocity in the axial direction and the maximum velocity in the channel, the highest currents are found, as expected, in the deep section of the channel, decreasing toward the sides and finally becoming zero at both shores.

The transverse shear gradient existing in such a flow structure gives rise, according to Delleur and McManus,[11] to another transverse circulation independent of that due to the earth's rotation. The current consists of two oppositely rotating spiral motions converging at the surface and diverging at the bed. The bed currents are considered to provide the forces causing the parabolically-shaped cross sections of natural channels.

This secondary motion, due to boundary-shear turbulence, like that due to the earth's rotation, has not been physically observed or measured in natural channels. It was assumed to exist in 1883 by Möller[12] and Stearns.[13] Delleur and McManus,[11] Einstein and Li,[14] and others[15,16] studied the motion analytically. Delleur traced the motion in the laboratory with moving pictures. Chebotarev[17] also reports laboratory investigations in which Losievskii observed similar double spiral motions in wide channels having a bed sloped toward the center.

There is a reasonable possibility that, in a straight section of a river, a second transverse motion is imposed on the axial flow in addition to that due to the earth's rotation. Since both transverse currents can be present only when a boundary exists, such as in a river, each should be an integral part of the system, so that one cannot be measured independently of the other in ordinary channels. Therefore under these circumstances, the deviations observed in natural channels could be larger or smaller than those computed for the rotation of the earth only, depending on the location in the cross section at which measurements are made.

Assume, for example, that the strength of the twin spiral cross currents on each side of the center of flow is similar in magnitude to that of the trans-

---

[11] Delleur, J. W., and McManus, D. S., "Secondary Flow in Straight Open Channels," Proceedings, 6th Midwestern Conference on Fluid Mechanics, University of Texas, Austin, Tex., 1959.

[12] Möller, Max, "Studien über die Bewegung des Wassers in Flüssen mit Bezugnahme auf die Ausbildung des Flussprofiles," Zeitschrift für Bauwesen, 1883, p. 201.

[13] Sterns, F. P., "On the Current-Meter, Together with a Reason Why the Maximum Velocity of Water Flowing in Open Channels is Below the Surface," Transactions, ASCE, Vol. XII, 1883, pp. 331-338.

[14] Einstein, H. A., and Li, Huon, "Secondary Currents in Straight Channels," Transactions, American Geophysical Union, Vol. 39, 1958.

[15] Tracy, H. J., "Turbulent Flow in a Three-Dimensional Channel," Journal of the Hydraulics Division, ASCE, Vol. 91, No. HY6, Proc. Paper 4530, Nov., 1965.

[16] Brundrette, E., and Baines, W. D., "The Production and Diffusion of Vorticity in Duct Flow," Journal of Fluid Mechanics, Vol. 19, Part 4, 1964, pp. 375-392.

[17] Chebotarev, A. I., Gidrologia Sushi ("Dry-Land Hydrology"), Gedromet Izdat, Leningrad, USSR, 1955 (in Russian).

verse motion imposed on the entire channel by the rotation of the earth. These assumptions are shown schematically on Figs. 7(*a*) and 7(*b*). A single transverse circulation composed of the two principle motions would result, similar to that shown in Fig. 7(*c*). The anticlockwise part of the circulation to the right of the center of flow would cease, while the left clockwise circulation would widen and gain momentum. In this way, a single cross circulation would result having its center shifted to the left and involving most of the channel. With this redistribution of transverse velocity there would be little or no transverse circulation on the right side to contribute resistance to the axial flow. Consequently, the area of maximum longitudinal velocity would be shifted to the right.

The field observations, Fig. 4, show that this type of circulation, a single transverse motion whose intensity is greater on the left side, occurred in the

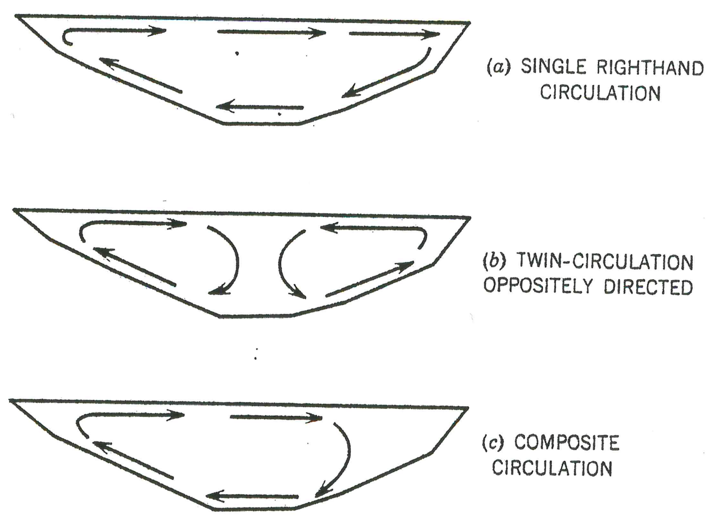

FIG. 7.—TRANSVERSE MOTION COMPOSED OF TWO TYPES OF TRANSVERSE CIRCULATION

river. On Fig. 6, it is further shown that the anticipated shift of the high velocity area to the right did occur.

In general, floating objects such as ice floes will be carried by the transverse circulation from the left side of the river toward the right side, where they will be transported downstream in the higher velocity current without being stranded on the right-hand shore. Particles that sink in water, e.g., material in a colloidal or flocculated state, will follow the circulation and possibly be deposited predominantly on the left-hand side. In this way the asymmetric cross sections of the channels, having greater depths and steeper slopes on the right side than on the left, are initiated.

There is good qualitative agreement between the observations and theory. The magnitude of the numerical results is in reasonable agreement if the various qualifications as described above are taken into account. The last of these, and the one which may have the greatest influence is, unfortunately, also the least proven, particularly with regard to direct observation and measurement. This, however, is one of the main features in describing the flow in open channels, for it is well known that there is no strictly accurate theory to describe turbulent flow and its related phenomena such as transverse circulation, even in simple rectangular channels.

CONCLUSIONS

1. Transverse circulation due to the earth's rotation is an inevitable feature of the flow in streams located in higher latitudes.
2. This circulation requires the presence of a boundary.
3. Its intensity relative to the axial flow depends, besides on the latitude, primarily on the numerical value of the ratio of the depth, d, to the mean current velocity, (v_o).
4. Based on the ratio d/v_o, rivers may be divided into two groups. In the first group ($d/v_o < 20$), represented generally by sediment-bearing rivers, the maximum deviation between surface and bed layers is less than 1° to 2°. In the second group ($d/v_o > 20$), characterized by large rivers like the St. Lawrence, river pool sections, and storage and power lakes, the deviations may be as much as 20°.
5. An asymmetry in the velocity distribution of the St. Lawrence indicated that a second transverse circulation was present, consisting of two oppositely rotating spirals generated by the transverse shear gradient.
6. Since both transverse currents occur because of the channel boundaries, neither can be measured independently under regular conditions.
7. Many factors of the phenomena are not fully understood. More observations and analytical investigations are required before the fundamental relationship can be formulated more exactly.

ACKNOWLEDGMENTS

The writer gratefully acknowledges the cordial cooperation of J. R. Clarke, who conducted part of the survey, and of E. S. Turner who helped to prepare the paper.

APPENDIX I.—ADDITIONAL REFERENCES

1. Gibson, A. H., Hydraulics and its Application, 3rd ed., D. Van Nostrand Co., Inc., Princeton, N. J., 1925.
2. Dantscher, K., "Die Flüsse und die Erdrotation," Wasserkraft und Wasserwirtschaft, Heft 12, 1942.
3. Vanoni, V. A., "Transportation of Suspended Sediment by Water," Transactions, ASCE, Vol. 111, 1946.
4. Shokry, Ahmed, "Flow Around Bends in an Open Flume," Transactions, ASCE, Vol. 115, 1950.
5. Kabelac, O. W., "Rivers under Influence of Terrestrial Rotation," Journal of the Waterways and Harbors Division, ASCE, Vol. 83, No. WW1, Proc. Paper 1208, Apr., 1957; with discussion by G. Tison, Vol. 83, No. WW3, Sept., 1957, pp. 1381-7—1381-11; and closure by O. W. Kabelac, Vol. 84, No. WW1, Jan., 1958, pp. 1523-7—1523-9.
6. Gabriel, V. G., "Influence of Coriolis Force on River Profiles," World Oil, Vol. 145, No. 4, Sept., 1957, p. 89.

APPENDIX II.—NOTATION

The following symbols are used in this paper:

a = Coriolis acceleration in horizontal plane;
a_x, a_y, a_z = components of Coriolis acceleration in X, Y, Z direction;
B = subscript, denotes bed;
b = width of channel;
C = chezy coefficient;
$C'_{(z)}$ = coriolis force per unit mass at height z;
D = constant for Eq. 23;
d = depth of water;
E_1, E_2 = functions in Eqs. 18, 19, and 22, the values of which can be derived from Fig. 2;
e = v/v_o = ratio between velocity in a particular layer and mean velocity;
F = frictional force per unit mass for turbulent motion;
F_x, F_y = components of frictional force in X and Y direction;
$G(z)$ = gravitational force at height z;
g = gravitational acceleration;
k = roughness length, related to the height of the irregularities at the bed;
k_o = Von Karman constant;
ℓ = Prandtl's "mixing length";
p = pressure force per unit mass from river bank;
r = radius of so-called "inertia circle";
r_c = radius of curvature of channel bend;
S = slope of water surface;
S_x, S_y = axial and transverse slope of water surface;
v = horizontal velocity of water particle;
v_x, v_y, v_z = velocity components in the X, Y, Z direction;
v_o = mean axial velocity along vertical;
$v_x(\eta), v_y(\eta)$ = axial and transverse velocity component at relative height $\eta = z/d$ above the bed;
x = horizontal coordinate in axial direction;
y = horizontal coordinate directed with the Coriolis force;
z = vertical coordinate;
α = angle of deviation from the axial direction of flow;
α_S = angle of deviation of the surface layer;
α_B = angle of deviation of layer near the bed;
β = a number related to the boundary friction;
δ = angle of inclination of transverse water surface;
ϵ = eddy viscosity;
η = relative or dimensionless height z/d;
ρ = density of water;
ϕ = geographical latitude; and
ω = angular velocity of the earth.